APPROXIMATE ANALYTICAL TEMPERATURE CALCULATION FOR A FUEL ROD WITH DEFECTS IN THE CONTACT LAYER

V. E. Minashin, V. N. Rumyantsev, and V. N. Levchenko

UDC 536.24

A solution has been obtained in infinite-series form for the two-dimensional thermal conduction equation for a rod with boundary conditions of the third kind and a heat-transfer coefficient varying around the perimeter.

This type of fuel rod (Fig. 1) consists of an active core 3, protective sheath 1, and a contact layer of fusible metal 2; defects of various types can arise, such as gas bubbles, breaks in the contact layer, or exfoliation, all of which lead to deterioration in the heat transfer from the core [1]. It is of some interest to examine the overheating arising from defects relative to the nominal temperature distribution. There are various analytical methods of solving this problem. For instance, a solution can be found [2] by variational methods, or else [3] as an infinite series meeting the boundary conditions at individual points.

We give below an approximate analytical method that has some similarity with that of [3], while having some differences: the method of transforming the boundary condition is novel, and the mode of solution construction is more logical.

The mathematical problem is then formulated as follows: the volume heat production rate in the core is constant in the height and cross section, while being zero in the sheath and contact layer. The physical parameters of all the materials are constant, while the heat-transfer coefficient for the cooled surface of the sheath is constant at all points. Also, the defects are infinitely long, and the axial and tangential components in the heat flux in the sheath are negligibly small by comparison with the radial component. The contact layer and sheath are incorporated in the problem as a thermal resistance by modifying the heat-transfer coefficient. Figure 2 shows the calculation scheme, where the two-dimensional steady-state temperature distribution is described by

$$\frac{1}{r}\frac{\partial}{\partial r}\left(\frac{\partial t}{\partial r}\right)+\frac{1}{r^2}\frac{\partial^2 t}{\partial \varphi^2}=-\frac{q_v}{\lambda_3} \tag{1}$$

and the boundary conditions by

$$-\lambda_3\frac{\partial t}{\partial r}=\alpha_e t \quad \text{for} \quad r=R \quad \text{and} \quad \varphi>\varphi_d, \tag{2}$$

$$-\lambda_3\frac{\partial t}{\partial r}=\alpha_d t \quad \text{for} \quad r=R \quad \text{and} \quad \varphi<\varphi_d, \tag{3}$$

where

$$\alpha_e=\frac{1}{\frac{R}{\lambda_2}\ln\left(\frac{R+\delta_2}{R}\right)+\frac{R}{\lambda_1}\ln\left(\frac{R+\delta_2+\delta_1}{R+\delta_2}\right)+\frac{1}{\alpha}\frac{R}{R+\delta_2+\delta_1}}, \tag{4}$$

$$\alpha_d=\frac{\alpha_K}{1+\frac{\alpha_K}{\alpha_e}}. \tag{5}$$

The rod temperature is reckoned from the mean temperature of the cooling surface.

Translated from Inzhenerno-Fizicheskii Zhurnal, Vol. 26, No. 3, pp. 509-515, March, 1974. Original article submitted March 14, 1973.

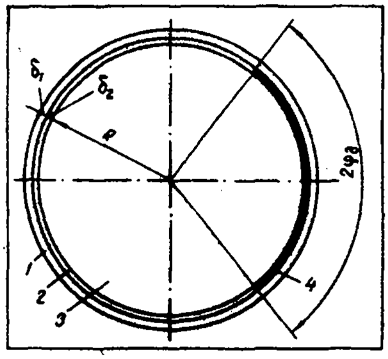

Fig. 1

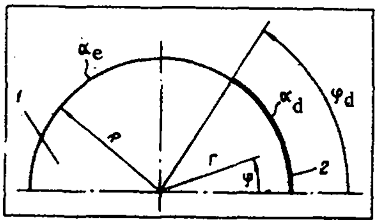

Fig. 2

Fig. 1. Cross section of rod with defect in contact layer: 1) sheath; 2) contact layer; 3) core; 4) defective zone.

Fig. 2. Calculation scheme for rod with defect: 1) core; 2) defective zone (impaired heat transfer).

The temperature distribution in the rod defect is sought as the sum of the nominal distribution t_n and the superheating ϑ due to the defect, i.e.,

$$t = t_n + \vartheta. \tag{6}$$

Then t_n is defined by

$$\frac{1}{r}\frac{\partial}{\partial r}\left(r\frac{\partial t_n}{\partial r}\right) = -\frac{q_v}{\lambda_3}, \tag{7}$$

$$-\lambda_3\frac{\partial t_n}{\partial r} = \alpha_e t_n \text{ for } r = R \text{ and any } \varphi. \tag{8}$$

The specific heat flux at the surface of the defect-free core is denoted by **q**:

$$\alpha_e t_n = q. \tag{9}$$

We substitute (6) into (1)-(3) and use (7)-(9) to get a system of equations for ϑ:

$$\frac{1}{r}\frac{\partial}{\partial r}\left(r\frac{\partial\vartheta}{\partial r}\right) + \frac{1}{r^2}\frac{\partial^2\vartheta}{\partial\varphi^2} = 0, \tag{10}$$

$$-\lambda_3\frac{\partial\vartheta}{\partial r} = \alpha_e\vartheta \text{ for } r = R \text{ and } \varphi > \varphi_d, \tag{11}$$

$$-\lambda_3\frac{\partial\vartheta}{\partial r} = \alpha_d\vartheta - (\alpha_e - \alpha_d)\frac{q}{\alpha_e} \text{ for } r = R,\ \varphi < \varphi_d. \tag{12}$$

Then under these conditions the task of finding the temperature distribution in the fuel rod amounts to finding the temperature distribution in a cylinder free from heat sources with in addition a constant heat flux near the defect, the heat being lost from the entire surface, but with different heat-transfer coefficients in the defective and perfect parts.

We perform an identical transformation on the boundary condition of (12) by adding and subtracting the expression for $\alpha_e\vartheta$:

$$-\lambda_3\frac{\partial\vartheta}{\partial r} = -q\left(1 - \frac{\alpha_d}{\alpha_e}\right)\left(1 + \frac{\vartheta}{q/\alpha_e}\right) + \alpha_e\vartheta; \tag{13}$$

on the basis that

$$1 - \frac{\alpha_d}{\alpha_e} = \frac{1}{1 + \dfrac{\alpha_K}{\alpha_e}}, \tag{14}$$

and from (13) we have

$$-\lambda_3\frac{\partial\vartheta}{\partial r} = \alpha_e\vartheta - q^*, \tag{15}$$

TABLE 1. Results for θ for $\alpha_k = 0$, $\varphi_d = \pi/2$, and $\varphi = 0$

No. of harmonics	$B = \frac{\alpha_e R}{\lambda_3}$							
	0,1	0,5	1,0	5,0	10,0	30,0	50,0	100,0
a_0	1,08	1,94	2,81	—	13,3	31,2	47,1	—
$a_0 + \sum_1^1 a_n \cos n\varphi$	1,19	1,91	2,74	8,47	15,0	40,0	64,7	—
$a_0 + \sum_1^2 a_n \cos n\varphi$	—	—	—	—	—	—	81,7	—
$a_0 + \sum_1^4 a_n \cos n\varphi$	—	—	—	—	—	—	87,4	—
$a_0 + \sum_1^5 a_n \cos n\varphi$	—	—	—	—	—	—	88,1	—
$a_0 + \sum_1^8 a_n \cos n\varphi$	1,20	1,99	2,96	10,4	19,4	54,7	89,7	—
$a_0 + \sum_1^{10} a_n \cos n\varphi$	—	—	—	—	—	—	89,8	177

where

$$q^* = q \frac{1 + \dfrac{\vartheta}{q/\alpha_e}}{1 + \alpha_k/\alpha_e}. \tag{16}$$

This transformation means that the heat is now transferred from the surface of the cylinder with a single heat-transfer coefficient; it is true that then the cylinder surface receives an unknown heat flux, which is a function of the unknown temperature.

We put (10), (11), and (15) in dimensionless form:

$$\frac{1}{\rho}\frac{\partial}{\partial\rho}\left(\rho\frac{\partial\theta}{\partial\rho}\right) + \frac{1}{\rho^2}\frac{\partial^2\theta}{\partial\varphi^2} = 0, \tag{17}$$

$$-\frac{\partial\theta}{\partial\rho} = B\theta \text{ for } \rho = 1 \text{ and } \varphi > \varphi_d, \tag{18}$$

$$-\frac{\partial\theta}{\partial\rho} = B\theta - B\frac{q^*}{q} \text{ for } \rho = 1 \text{ and } \varphi < \varphi_d, \tag{19}$$

where

$$\theta = \frac{\vartheta}{q/\alpha_e};\quad B = \frac{\alpha_e R}{\lambda_3};\quad \rho = \frac{r}{R};\quad \frac{q^*}{q} = \frac{1 + \theta}{1 + \dfrac{\alpha_k}{\alpha_e}}.$$

We expand the unknown heat flux q^*/q as a Fourier series:

$$\frac{q^*}{q} = a_0 + \sum_{n=1}^{\infty} a_n \cos n\frac{K}{2}\varphi, \tag{20}$$

where $K = \pi/\varphi_d$, and we derive the distribution of the temperature θ (Fig. 3) first of all from the first term in (20) and then from the second, and so on.

The distribution of θ arising from the constant component a_0 is found from (17)-(19), with (19) taking the form

$$-\frac{\partial\theta}{\partial\rho} = B\theta - a_0 B. \tag{21}$$

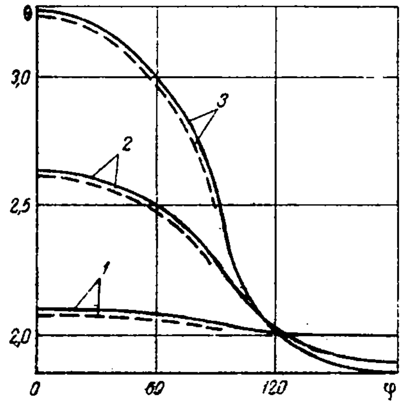

Fig. 3. Surface temperature of core for $\varphi_d = \pi/2$ and $\alpha_e R/\lambda_3$ of: 1) 0.077; 2) −0.622; 3) −1.23. The broken lines are from analytical calculation, while the solid lines are from numerical solution.

This problem is solved by separating the variables

$$\theta(a_0) = a_0 \left\{ \frac{\varphi_d}{\pi} + \frac{2B}{\pi} \sum_{m=1}^{\infty} \frac{\sin m\varphi_d}{m(m+B)} \cos m\varphi \right\}. \tag{22}$$

We find the distribution of θ due to the second term in (20) by variable separation using the conditions of (17)-(19); the boundary condition of (19) takes the form

$$-\frac{\partial\theta}{\partial\rho} = B\theta - Ba_n \cos n\frac{K}{2}\varphi. \tag{23}$$

We get

$$\theta\left(a_n \cos n\frac{K}{2}\varphi\right) = \frac{a_n}{\pi}\,\frac{2\sin n\frac{\pi}{2}}{nK} + \frac{2a_nB}{K(nK+2B)}\cos n\frac{K}{2}\varphi$$
$$+\frac{a_nB}{\pi}\sum_{m=1}^{\infty}\left\{\frac{\sin\left(n\frac{K}{2}-m\right)\frac{\pi}{K}}{\left(n\frac{K}{2}-m\right)} + \frac{\sin\left(n\frac{K}{2}+m\right)\frac{\pi}{K}}{n\frac{K}{2}+m}\right\}\frac{\cos m\varphi}{m+B}, \tag{24}$$

apart from m = nK/2, where m is an integer.

We combine (22) and (24) to get the superheating of the defective rod in terms of the unknown Fourier coefficients:

$$\theta = a_0\left\{\frac{1}{K} + \frac{2B}{\pi}\sum_{m=1}^{\infty}\frac{\sin m\frac{\pi}{K}}{m(m+B)}\right\} + \sum_{n=1}^{\infty}\left\{\frac{a_n}{\pi}\,\frac{2\sin n\frac{\pi}{2}}{nK}\right.$$
$$\left.+\frac{2a_nB}{K(nK+2B)}\cos n\frac{K}{2}\varphi + \frac{a_nB}{\pi}\sum_{m=1}^{\infty}\left[\frac{\sin\left(n\frac{K}{2}-m\right)\frac{\pi}{K}}{n\frac{K}{2}-m} + \frac{\sin\left(n\frac{K}{2}+m\right)}{n\frac{K}{2}+m}\right]\frac{\cos m\varphi}{m+B}\right\}. \tag{25}$$

If the size of the defect is such that m = nK/2 is impossible, then the term $[2a_nB/K(nK+2B)]\cos n(K/2)\varphi$ should be omitted.

We find the unknown coefficients a_0 and a_n from the solution to a system of p + 1 equations, which is derived from

$$\frac{q^*}{q} = \frac{1+\theta}{1+\frac{\alpha_K}{\alpha_e}} = a_0 + \sum_{n=1}^{p} a_n \cos n\frac{K}{2}\varphi,$$

and satisfy this with p + 1 different values for φ, i.e., for $\varphi = \varphi_i = i(\varphi_d/p)$, where i = 0-p and p is the number of harmonics in the expansion of the unknown heat flux.

Table 1 gives results on θ derived with an M-20 computer for α_K of 0 and φ_d of $\pi/2$; it is clear that it is sufficient to take 8-10 harmonics in order to obtain reasonably accurate results for values of B ranging from 0.1 to 100.

For comparison, we calculated the temperature distributions for the same conditions for α_k of 0 and φ_d of $\pi/2$ by numerical methods using an M-20 computer.

The results show that this method of calculating the temperature distributions in rods with defects allows one to determine this for a range in $\alpha_e R/\lambda_3$ from 0.01 to 100 with an accuracy sufficient for engineering calculations.

This method also enables one to calculate a three-dimensional temperature distribution due to a defect of finite size, but the working formulas are rather lengthy.

NOTATION

r, φ, t	are the current radius, angle, and temperature calculated from mean liquid temperature;
φ_d	is the angle determining dimensions of defective part of perimeter;
R, δ_1, δ_2	are the radius of core, jacket thickness, and contact layer thickness;
$\lambda_1, \lambda_2, \lambda_3$	are the thermal conductivity of materials of jacket, contact layer, and core;
α	is the heat-transfer coefficient between fuel rod and cooling liquid;
α_e	is the equivalent heat-transfer coefficient between liquid and sound part of core perimeter, incorporating thermal conductivity of contact layer and jacket;
α_k	is the thermal conductivity of defect;
α_d	is the equivalent heat-transfer coefficient between liquid and defective part of core perimeter, incorporating thermal conductivity of contact layer of jacket and defect;
$B = \alpha_e R/\lambda_3$	a dimensionless complex number;
q_v	is the specific heat production rate.

LITERATURE CITED

1. A. S. Zaimovskii et al., Reactor Fuel Rods [in Russian], Atomizdat, Moscow (1966).
2. E. S. Turilina and I. D. Voskresenskii, Teplofizika Vyssokikh Temperatur, 4, No. 5, 660 (1966).
3. R. Nijsing, Nucl. Eng. and Design, 4, No. 1, 1 (1966).